20+20+20+20+10=90

(1/2)

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Thermal Physics - T1

1. $V = V_0 = 1500\,m^3$ per year

20pt · CH_4 : $M = 4 \cdot M_H + M_C = 16\,g/mol = 16 \cdot 10^{-3}\,kg/mol$ 10!

$m = ?\,kg$

$T = 20°C = 293.15\,K$ $pV = N k_B T = n_m RT$

$p = 1\,bar = 10^5\,Pa$

$n_m = ?\,mol$ $k_B = 1.381 \cdot 10^{-23}\,J/K$

$\frac{V_l}{V_0} = ?$ if $T_l = 15°C = 288.15\,K$ $N_A = 6.02 \cdot 10^{23}\,mol^{-1}$

$$N = \frac{pV}{k_B T}$$

$$N = \frac{10^5 \cdot 1500}{1.381 \cdot 10^{-23} \cdot 293.15}$$

$N \doteq 3.705 \cdot 10^{28}$ molecules $\Rightarrow n_m = \frac{N}{N_A} = \frac{3.705 \cdot 10^{28}}{6.02 \cdot 10^{23}} \doteq 6.15 \cdot 10^4\,mol$

$m = M n_m = 16 \cdot 10^{-3}\,\frac{kg}{mol} \cdot 6.15 \cdot 10^4\,mol = 984\,kg$

At $T = 20°C$ and $p = 1\,bar$, methane of volume $1500\,m^3$ corresponds to $6.15 \times 10^4\,mol$ or $984\,kg$.

$\frac{V}{T} = \frac{N k_B}{p} = const.$

$\frac{V_0}{T_0} = \frac{V_l}{T_l} \Rightarrow V_l = \frac{T_l}{T_0} V_0$

$V_l = \frac{288.15\,K}{293.15\,K} \cdot 1500\,m^3$

$V_l \doteq 1474\,m^3 = 0.983\,V_0 = 98.3\%\,V_0$

- if gas was kept at temperature 15°C, the consumed volume would drop by 1.7% to $1474 m^3$.

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20 pt.

2. $50e^-$: ↑ or ↓

$25\uparrow : 25\downarrow$

$24\uparrow : 26\downarrow$

$1\uparrow : 49\downarrow$

The outcome of $25\uparrow : 25\downarrow$ is significantly more likely than $1\uparrow : 49\downarrow$ because the macrostate $25\uparrow : 25\downarrow$ corresponds to larger number of microstates, whereas $1\uparrow : 49\downarrow$ has only small number of microstates. $24\uparrow : 26\downarrow$ will be close to $25\uparrow : 25\downarrow$.

Total number of microstates: $2^{50} \approx 1.13 \cdot 10^{15}$

↳ each e^- can be in either of 2 states and all combinations are possible in a total of 50 electrons. Each microstate is same likely, has prob. $\frac{1}{2^{50}}$

Macrostate $1\uparrow : 49\downarrow$ corresponds to 50 microstates – we require one e^- to be up and 49 down. There are 50 e^- which could be up while the rest is down. These 50 states are indistinguishable as we are not able to distinguish electrons from each other.

~~Proba~~ Probability $P(1\uparrow : 49\downarrow) = \frac{50}{2^{50}} = \frac{50}{1.13 \cdot 10^{15}} = \underline{4.42 \cdot 10^{-14}}$

Macrostate $24\uparrow : 26\downarrow$ means that there are 24 electrons up and 26 down which are not distinguishable. The number of microstates is given by $\binom{50}{26} = \binom{50}{24} = \frac{50!}{24!\,26!} = 1.215 \cdot 10^{14}$

Therefore its probability is $P(24\uparrow : 26\downarrow) = \frac{1.215 \cdot 10^{14}}{1.13 \cdot 10^{15}} \doteq 0.108 = \underline{10.8\%}$

This is significantly more than for $1\uparrow : 49\downarrow$

Macrostate $25\uparrow : 25\downarrow$ is the most probable macrostate as it corresponds to the highest number of microstates. ~~This~~ Them (The number of microstates) is given by

$\binom{50}{25} = \frac{50!}{25!\,25!} \doteq 1.264 \cdot 10^{14}$

Therefore its probability is $P(25\uparrow : 25\downarrow) = \frac{1.264 \cdot 10^{14}}{1.13 \cdot 10^{15}} \doteq 0.112 = \underline{11.2\%}$

The difference between the probability of outcomes $(25\uparrow : 25\downarrow)$ and $(1\uparrow : 49\downarrow)$ is about 13 orders of magnitude.

8.

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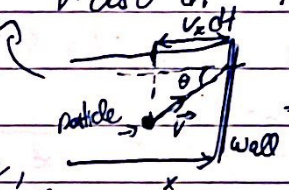
20 pt.

3. Particles effusing through a small hole are the same particles as those which would hit the wall in the same place if there was no hole.

The ~~number~~ fraction of particles hitting a wall of unit area A can be calculated as the fraction of particles travelling to the wall ~~on (their angle~~ under angle*K $\in (\theta, \theta + d\theta) = \frac{1}{2}\sin\theta\, d\theta$ multiplied by the fraction ~~particles~~ of Maxwell-Boltzmann particles with speeds $\in (v, v+dv) = n f(v)\, dv$ times the ~~number fraction of~~ fraction of part. in volume from which the particles can reach the wall in ~~dt~~ time $dt = v\cdot\cos\theta\, dt$.

This yields the result of:

$A \cdot v \cdot \cos\theta\, dt\; n f(v)\, dv \cdot \frac{1}{2}\sin\theta\, d\theta$



$v_x = v\cdot\cos\theta$

$\Rightarrow v_x dt = v\cdot\cos\theta\, dt$

expressing this per unit area and unit time, we get: $v\cos\theta\, n f(v)\, dv\, \frac{1}{2}\sin\theta\, d\theta = \underline{\underline{\frac{n}{2} v f(v) dv \cos\theta \sin\theta\, d\theta}}$

$f(v)$ is Maxwell-Boltzmann* distributed however the expression for effusing particles has additional v ~~term~~, hence $\propto v^3 e^{-mv^2/2k_BT}$ and is not Maxwell-Boltzmann distributed.

The mean velocity of effusing particles is given by:

$$\langle v\rangle = \frac{\frac{n}{2}\int_0^\infty v^2 f(v)dv \int_0^{\pi/2}\cos\theta\sin\theta d\theta}{\frac{n}{2}\int_0^\infty v f(v)dv \int_0^{\pi/2}\cos\theta\sin\theta d\theta} = \frac{\int_0^\infty v^2 f(v)\,dv}{\int_0^\infty v\, f(v)dv} \overset{*}{=} \frac{A\int_0^\infty v^4 e^{-mv^2/2k_BT}dv}{A\int_0^\infty v^3 e^{-mv^2/2k_BT}dv} =$$

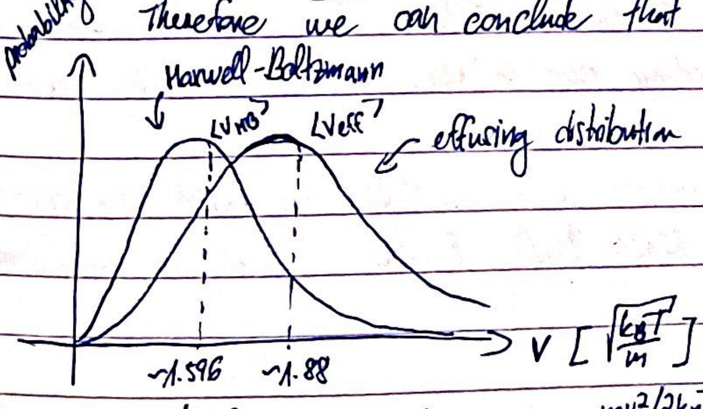
partition function to normalise

$$= \frac{\frac{1}{2}\cdot\frac{3}{4}\sqrt{\frac{\pi}{\left(\frac{m}{2k_BT}\right)^5}}}{\frac{1}{2\left(\frac{m}{2k_BT}\right)^2}} = \frac{3}{4}\sqrt{\frac{\pi}{\frac{m}{2k_BT}}}\,\frac{\left(\frac{m}{2k_BT}\right)^2}{\left(\frac{m}{2k_BT}\right)^2} = \underline{\frac{3}{4}\sqrt{\frac{2\pi k_BT}{m}}}.$$

$$= \underline{\underline{\langle v_{eff}\rangle = 3\sqrt{\frac{\pi k_BT}{8m}}}} \quad \text{for effusing particles}$$

$\approx 1.88\sqrt{\frac{k_BT}{m}}$ whereas for MB distributed: $\langle v\rangle_{MB} = \sqrt{\frac{8k_BT}{\pi m}} \approx 1.596\sqrt{\frac{k_BT}{m}}$

Therefore we can conclude that effusion selects faster particles.



*$f(v) \propto v^2 e^{-mv^2/2k_BT} \Rightarrow f(v) = A v^2 e^{-mv^2/2k_BT}$

normalisation constant

K this angle is measured from the normal of wall

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! Question 5 is on second sheet !

20 pt.

4. $E_0 = 0eV$ $\quad \frac{N_0}{N_1} = e^{-(E_0-E_1)/k_BT} = e^{-(0-4.8\cdot10^{-19})/k_BT} = e^{4.8\cdot10^{-19}/k_BT}$

$E_1 = 3eV \doteq 4.8\cdot10^{-19}J$ $\quad = e^{4.8\cdot10^{-19}/1.381\cdot10^{-23}T} = e^{3.48\cdot10^{4}/T}$

$\cancel{\frac{N_0}{N_1} = ?}$

a) $T = 10^0K = 1K$: $\frac{N_0}{N_1} = e^{3.48\cdot10^4/10^0} = e^{3.48\cdot10^4} = e^{34800}$

N_0 = # particles in ground state

N_1 = # particles in excited state

N = total # particles

b) $T = 10^2K$: $\frac{N_0}{N_1} = e^{3.48\cdot10^4/10^2} = e^{3.48\cdot10^2} = e^{348}$

c) $T = 10^4K$: $\frac{N_0}{N_1} = e^{3.48\cdot10^4/10^4} = e^{3.48} \doteq 32.46$

d) $T = 10^6K$: $\frac{N_0}{N_1} = e^{3.48\cdot10^4/10^6} = e^{0.0348} \doteq 1.035$

$$\frac{N_0}{N} = ? = \frac{e^{-E_0/k_BT}}{Z} = \frac{1}{Z} = \frac{1}{1+e^{-3.48\cdot10^4/T}}$$

$$\frac{N_1}{N} = ? = \frac{e^{-E_1/k_BT}}{Z} = \frac{e^{-3.48\cdot10^4/T}}{Z} = \frac{e^{-3.48\cdot10^4/T}}{1+e^{-3.48\cdot10^4/T}}$$

where $Z = e^{-E_0/k_BT} + e^{-E_1/k_BT} = 1 + e^{-4.8\cdot10^{-19}/1.38\cdot10^{-23}T} = e^{-3.48\cdot10^4/T}$

a) $T = 10^0K$: $\frac{N_0}{N} = \frac{1}{1+e^{-3.48\cdot10^4}}$, $\frac{N_1}{N} = \frac{e^{-3.48\cdot10^4}}{1+e^{-3.48\cdot10^4}}$

b) $T = 10^2K$: $\frac{N_0}{N} = \frac{1}{1+e^{-3.48\cdot10^2}} = \frac{1}{1+e^{-348}}$, $\frac{N_1}{N} = \frac{e^{-348}}{1+e^{-348}}$

$\frac{N_1}{N}$: very small numbers

c) $T = 10^4K$: $\frac{N_0}{N} = \frac{1}{1+e^{-3.48}} \doteq 0.9701$, $\frac{N_1}{N} = \frac{e^{-3.48}}{1+e^{-3.48}} \doteq 0.0299$

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d) $T = 10^6K$: $\frac{N_0}{N} = \frac{1}{1+e^{-3.48\cdot10^{-2}}} \doteq 0.5087$, $\frac{N_1}{N} = \frac{e^{-3.48\cdot10^{-2}}}{1+e^{-3.48\cdot10^{-2}}} \doteq 0.4913$

⇒ gas at room temperature is slightly ~~the~~ below 300K, so of the 10^2 ~~and~~ order of magnitude. barely ~~the~~ any molecules and atoms will be excited in this range. Atoms don't really get excited

⇒ plasma at the surface of sun is about 6000K, close 10^4K above. At these temperatures, some of the atoms get excited, however about 97% remains in ground state.

⇒ core of the Sun has temperatures near 10^6K. At these temperatures, excitation commonly occurs. Nearly half of the atoms or molecules will be ~~an~~ excited.

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10 p.

5. $\frac{1}{k_B T} = \frac{d \ln(\Omega(E))}{dE}$, where T is the temperature, $\Omega(E)$ is the number of microstates at energy E of the system

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